ESC195 Week 9 Notes

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Equation of a Tangent Plane: Suppose that f has continuous partial derivatives. An equatio nfo the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is:

$$\mathbf{z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

Linear Functional Representation of Tangent Plane:

$$\mathbf{L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)}$$

where:
a, b is a particular point on the original function, f, and x, y can be varied to find the height of the plane $(L(x, y))$ at that particular point.

Linear Approximation (Tangent Plane Approximation:)

$$\mathbf{f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)}$$

Differentiability of f: If the partial derivatives f_x and f_y exist near (a, b) and are continuous at (a, b), then$f$ is differentiable at (a, b).

Partial Differentials:

$$\mathbf{dz = f_x(x, y)dx + f_y(x, y)dy}$$

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Partial Chain Rule (Case 1): Suppose that $z = f(x, y)$ is a differentiable function of x and y where $x = g(t)$ and $y = h(t)$ are both differentiable function s of t. Then z is a differentiable function of t and:

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

The Chain Rule (Case 2): Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t. Then:

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Chain Rule for Implicit Differentiation:

$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{F_x}{F_y}$$

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Directional Derivatives: If f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $\mathbf{u} = \langle a, b \rangle$

$$\mathbf{D_u f(x, y) = f_x(x, y)a + f_y(x, y)b}$$

$$\mathbf{D_u f(x, y) = f_x(x, y)cos}\theta \mathbf{+ f_y(x, y)sin}\theta$$

Gradient: If f is a function of two variables, x and y, then the **gradient** of f is the vector function ∇f defined by:

$$\mathbf{\nabla f(x, y) = \langle \; f_x(x, y) \; , \quad f_y(x, y) \; \rangle}$$

Alternative Directional Derivative Expression:

$$D_u f(x, y) = \nabla f(x, y) \cdot \mathbf{u}$$

Gradient 3D: If f is a function of three variables, x, y, and z then the **gradient** of f is the vector function ∇f defined by:

$$\mathbf{\nabla f(x, y, z) = \langle \; f_x(x, y, z) \; , \quad f_y(x, y, z) \quad f_z(x, y, z) \; \rangle}$$

or:

$$\mathbf{\nabla f = \langle f_x, f_y, f_z \rangle}$$

therefore:

$$D_u f(x, y, z) = \nabla f(x, y, z) \cdot \mathbf{u}$$

Maximum Directional Derivatives: The maximum value of the directional derivative $D_u f(x)$ is $|\nabla f(\mathbf{x})|$ and it occurs when $\mathbf{u}$ has the same direction as the gradient vector $\nabla f(x)$.

In other words, the maximum value of the directional derivative is like picking some point on our surface, (pretend its hilly terrain), and asking the question, if we walk in some direction from our current point, which direction will be steepest?

Tangent Planes to Level Surfaces: Suppose S is a surface with equation $F(x, y, z) = k$, that is, it is a level surface of a function F of three variables. the **tangent plane to the level surface is:**

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

Properties of the Gradient Vector: let f be a differentiable function of two or three variables and suppose that $\nabla f(\mathbf{x} \neq \mathbf{0})$:

- The directional derivative of f at $\mathbf{x}$ in the direction of a unit vector $\mathbf{u}$ is given by $D_u f(\mathbf{x}) = \nabla f(\mathbf{x}) \cdot \mathbf{u}$
- $\nabla f(\mathbf{x})$ points in the direction of maximum rate of increase of f at $\mathbf{x}$, and that maximum rate of change is $|\nabla f(\mathbf{x})|$
- $\nabla f(\mathbf{x})$ is perpendicular to the level curve or level surface of f through $\mathbf{x}$